easily studied if the parameters of the first wave are known. Fig.3 shows the wave pattern for such cases.

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FORCED OSCILLATIONS IN IMPERFECT AND STATICALLY LOADED SHELLS*

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The influence of small, non-axisymmetric imperfections in the middle surface and of a static load, on the amplitude of the forced oscillations of shells of revolution of zero curvature, is studied. For this purpose, shells acted upon by a mixed load, namely a static and dynamic load, are computed. The problem of a mixed load applied to an ideal shell is reduced to the problem of statics for a shell containing imperfections which vary with time. The amplitude-frequency relations are constructed for the flexure of statically loaded shells within the range of the lowest resonance frequencies. It is shown that in the case of statically loaded shells these relations differ essentially from those for load-free shells. The greatest increase in the amplitude of forced oscillations is observed in forms where the number of waves in parallel corresponds to the lowest frequencies.

In investigating the influence of static loads or form imperfections on the dynamic behaviour of shells, the greatest attention has been given, as a rule, to the change in the resonance frequencies. In practice, it is essential to know the behaviour of the oscillation amplitude under static loads, or resulting from form imperfections, and this is important when studying the dynamic behaviour of loaded shells as a whole.

One of the methods of solving the problem of the statics or dynamics of imperfect shells is based on the introduction of irregularity parameters into the initial system of equations. A linear system of equations is chosen as the initial system. The small-parameter method is then used, just as was done in problems of the statics of imperfect shells /1-3/. The same approach can be used in the problem of shells under a mixed load, and such a problem has been studied experimentally**.(**Solodilov V.E. Study of the natural oscillations of shells using holographic interferometry. Candidate Dissertation, Moscow, Inst. problem mechaniki, Akad. Nauk SSSR, 1980).

Let us consider the forced oscillations of a shell of revolution with an imperfect middle surface, excited by an axisymmetric harmonic load. We shall describe the imperfections in the middle surface of the shell using functions of the type $w_0 = \varepsilon r(z) \cos m\varphi$ where w_0 is the initial sag, z is the meridianal coordinate, φ is the circular coordinate, m is the number of waves in parallel, and ε is a number, small compared with the relative thickness of the shell. We shall write the coefficients of the solution of the system of equations describing the forced oscillations of an arbitrary shall, in the form of series in powers of the small parameter ε. After substituting the coefficients and the solution into the initial system, the latter splits into several subsystems. The zeroth approximation corresponds to the problem of the forced oscillations of a perfect shell of revolution. Every subsequent approximation is constructed by integrating the system of equations for the perfect shell of revolution, with various right-hand sides in the equations of equilibrium as well as in the geometrical relations. Thus the analysis of a shell with small, non-axisymmetric imperfections,

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reduces to a numer of calculations of a perfect shell.

Confining ourselves to terms that are linear in ε, we obtain two dependent systems of equations. When the shell is conical, the second system of equations for constructing the first approximation has the form

$$
\begin{aligned}
& L_i(T_j^1, S^1, N_j^1, G_j^1, H^1) = Z_i \qquad (1)\\
& i = 1, 2, \ldots, 5;\ j = 1, 2\\
& Z_1 = \sin\alpha\,(r\sin\alpha + Rr'\cos\alpha)\frac{dT_1^0}{dz} - (r'R - p\sin^2\alpha)(T_1^0 - T_2^0) -\\
& \quad \cos^2\alpha\,\frac{dp}{dz}N_1^0\\
& Z_2 = m\,[r'R\cos\alpha N_1^0 - \sin\alpha rR'(T_1^0 - T_2^0)/R]\\
& Z_3 = \cos^2\alpha\,\frac{dp}{dz}T_1^0 + p\sin\alpha\cos\alpha\,\frac{dN_1^0}{dz} - C_1T_2^0 - C_2\frac{N_1^0}{\cos\alpha}\\
& Z_4 = \frac{p}{R}(G_2^0 - G_1^0) - r\frac{dG_1^0}{dz} + \left(p\sin\alpha + \frac{r}{\cos\alpha}\right)N_1^0\\
& Z_5 = mp\sin\alpha\,(G_1^0 - G_2^0)/R
\end{aligned}
$$

are the equations of equilibrium, (L_i are the known operators of the equations of equilibrium of the shells, /4/ Z_i are the right-hand sides of the equations of equilibrium representing the load), and

$$
\begin{aligned}
& \varepsilon_1^1 = \varepsilon_1 + \cos^2\alpha\left(\frac{dp}{dz}\cos\alpha w^0 - p\sin\alpha\frac{du_1^0}{dz}\right) \qquad (2)\\
& \varepsilon_2^1 = \varepsilon_2 + C_2u_1^0 - \cos\alpha C_1w^0\\
& \omega^1 = \omega + m\cos\alpha\,(p\sin\alpha u_1^0/R - 2r'\cos\alpha w^0)\\
& \gamma_1^1 = \gamma_1 + \cos^2\alpha\left(\cos\alpha\frac{dp}{dz}u_1^0 + \sin\alpha p\frac{dw^0}{dz}\right)\\
& \gamma_2^1 = \gamma_2 - mr'\cos^2\alpha u_1^0,\quad \varkappa_2^1 = \varkappa_2 - C_2\gamma_1^0\\
& \tau^1 = \tau + m\cos\alpha\,(r'\cos\alpha\varepsilon_2^0 - p\sin\alpha\gamma_1^0/R)\\
& \varkappa_1^1 = \varkappa_1 + \sin\alpha\cos^2\alpha p\frac{d\gamma_1^0}{dz},\quad p = (rR)'\\
& C_1 = [(m^2 - 1)\,r - p\sin\alpha\cos\alpha]/R,\quad C_2 = (r'R\cos^3\alpha - r\sin^3\alpha)/R
\end{aligned}
$$

are geometrical relations.

The deviation from axial symmetry was specified by the variable distance a between the axis of revolution and the middle surface of the shell

$$a = R\,(1 + w_0),\ R = R_0\,(1 + r\,\mathrm{tg}\,\alpha) \qquad (3)$$

Here R_0, α is the radius of the smaller face and half-angle of the cone, and u_j, w, ε_j, ω, γ_j, $\varkappa_j$, τ, T_j, S, N_j, G_j are the displacements, deformation and force factors in the notation of /4/. The superscript indicates the zeroth or the first approximation. The deformations and angles of rotation without a superscript are known geometrical relations resulting from the non-axisymmetric force load Z_i. The elasticity relations are not given here, since they also remain homogeneous to a first approximation. We note that when $\alpha = 0$, the system (1)-(3) holds for a cylindrical shell. The above system of equations was integrated using Godunov's method.

We studied the forced oscillations of a cylindrical shell with initial sag of the form (3). The shell dimensions were: thickness $h = 0.5$ mm, radius $R_0 = 100$ mm, and length $L = 400$ mm. The intial sag along the meridian was of the form $r(z) = \sin\pi z/L$. A uniform external pressure q was chosen as the load, harmonic with respect to time $q = q_0e^{i\omega t}$, $q_0/(2E) = 3.75\cdot10^{-6}$ (E is Young's modulus and ω is the angular frequency of oscillations). The shell edges were assumed clamped, and oscillations of frequency $f = \omega/(2\pi) = 483$ Hz were studied. This frequency is close to the lowest characteristic frequency of the shell to which the form with $m = 5$ waves in parallel corresponds.

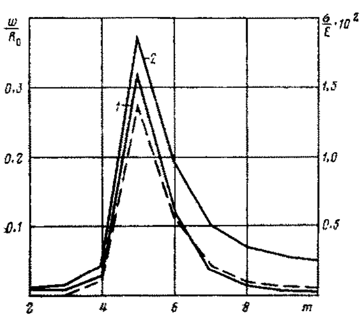

Fig.1

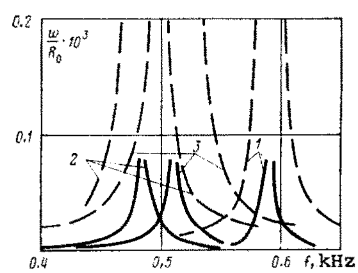

Fig.2

Various forms of imperfections of the type (3), obtained by varying m from 2 to 10, were studied, and the magnitude of the small parameter was assumed to be $\varepsilon = h/(4R_0)$. Fig.1 shows the dependence of the largest values of the flexure amplitude w (the dashed line) and the stress amplitudes (the solid lines) on m. Curve *1* corresponds to the stresses caused by tangential forces, and curve *2* to the stresses due to the moments. The effect of the imperfections with $m=2, 3, 4$ waves along the parallel is relatively weak. The imperfections with $m=5$ waves along the parallel show a substantial influence on the displacements and on the stress-deformation state of the shell. The displacements in the imperfect shell exceed those of the perfect shell by two orders of magnitude, and the stresses due to moments exceed those caused by tangential forces. The effect of the imperfections becomes gradually weaker as m increases, although it still remains considerable. The qualitative nature of the results shown in Fig.1 should be noted, especially that with $m=5$, where the frequency of the load responsible for the forced oscillations is close to the characteristic frequency of the perfect shell.

We have also studied the forced oscillations of perfect, statically loaded shells. The static load was axisymmetric. The dynamic load leads to non-axisymmetric flexures, which in this case play the part of imperfections. The imperfections vary with time and represent a type of forced oscillations of a perfect shell, i.e (3) w_0 will represent the form of the forced oscillations of the perfect shell. Then the problem of forced oscillations of a perfect, statically loaded shell, can be considered as a problem of statics for the shells with imperfections varying with time.

Let us apply the method of expansion in terms of a small parameter to this problem. We choose, as the small parameter, the maximum value of the flexure amplitude of a perfect shell. Then the zeroth-order approximation will be obtained by solving the axisymmetric problem of statics for a perfect shell. The first-order approximation will be determined by two terms. The first term will represent the solution of the non-axisymmetric problem of the forced oscillations of a perfect shell under a real dynamic load. The second term will be characterized by the interaction of the statics and dynamics, and will be constructed by solving these problems, with the right-hand sides of the first-order approximation playing the part of the load.

The general solution written e.g. for the flexure w in the problem of forced oscillations of a perfect shell under a static load, has the form

$$w = w_1 + [w_2 + w_*]\, e^{i(m\varphi+\omega t)} \tag{4}$$

where w is the total flexure, w_1 is the flexure in the axisymmetric problem of statics for a perfect shell, w_2 is the form of the oscillations of a perfect shell under a dynamic load, and w_* is the solution of the problem of statics for a shell with dynamic imperfections, i.e. the solution of the system (1), (2).

It should be noted that w_2 is small compared with the shell thickness. This means that the method cannot be used near the resonance where w_2 is large, and expansion in terms of a small parameter cannot be carried out.

We have studied the behaviour of the clamped cylindrical shell mentioned above, acted upon by a mixed load. The static load consisted of a uniform axisymmetric pressure $q_0/(2E) = 3.75\cdot10^{-8}$, and the dynamic load was

$$q = 0.01\,(1 - x^2)^2\,(0,\ 236 + 0.491x + 0.656\,x^2)\, q_0\, e^{i(m\varphi+\omega t)}$$

where $x = z/R_0, m = 4, 5, 6$, i.e. we studied the oscillations in the region of lower characteristic frequencies. Fig.2 shows amplitude-frequency graphs for the flexure w of the shell for $m=4, 5, 6$. The solid line refers to the dynamic load only, and the dashed lines to the mixed loads. The numbers 1, 2, 3 correspond to $m=4, 5, 6$. In the case of a shell under a mixed load, the above relations are constructed away from resonance, so that the quantity w_2 is small compared with the shell thickness.

We see from the graphs that the relations constructed for $q=0$ and $q\neq 0$ differ substantially from each other. Computations have shown that a uniform static load appreciably increases the dynamic oscillations of the shell at harmonics $m=4, 5, 6$. The greatest effect was observed in oscillations with $m=5,6$ waves along a parallel. The lowest characteristic frequency of the shell corresponds to the form of the oscillations with $m=5$. Therefore the greatest increase in the flexure amplitude was observed when the form of the forced oscillations corresponded to the characteristic frequency of the shell. In the case of oscillations with $m=5, 6$, the flexure amplitude exceeded the flexure caused by a purely dynamic load, by an order of magnitude.

The method of holographic interferometry (see the footnote) was used to study the behaviour of a perfect truncated conical shell acted upon by an axisymmetric compressive force and a dynamic, non-axisymmetric load. It was found that the magnitude of the flexure amplitude in the resonance forms depends essentially on the magnitude of the static load, practically from the instant the load is applied. When the compressive force was increased, the oscillation amplitudes increased appreciably, although the dynamic load remained constant. In the initial loading stages the dependence of the flexure amplitude on the compressive force was practically linear, and this confirms the linear dependence of the approximate solution obtained on the static load in formula (4).

It was precisely such a shell that was investigated. Its dimensions were: thickness $h = 0.7$ mm, half-angle $\alpha = 18.5°$, length of the axis of revolution $L = 104$ mm. and the radius of the smaller face $R_0 = 40$ mm. The shell was clamped by its larger face, and the smaller face was joined to a prefectly rigid nut.

The table below shows the characteristic frequencies in Hz obtained numerically for various values of m and n, (n is the number of the frequency for fixed m). The values obtained agree almost exactly with those obtained by Solodilov.

Next, forced oscillations of the same shell were studied under a static load. The static load was represented by a compressive force acting on the smaller face through the rigid nut. The dynamic load remained the same as in the case of the cylindrical shell.

Table

n	$m = 4$	5	6	7
1	2981	2577	2619	2965
2	8315	4837	4422	4460

The dependence of the total flexure on the compressive force was investigated. When only the first-order approximation is taken into account, the dependence is linear. Using the same static load in all cases, we studied various types of dynamic load differing from each other by the number m and the frequency. The frequency was chosen as before, as close to the natural frequency as was allowed by the method of expansion in terms of the small parameter. The greatest increase in the flexure was observed at the first frequencies $(n = 1)$ with $m = 5$ waves along the parallel, and at the second frequencies $(n = 2)$ with $m = 6$. According to the table the oscillations correspond to the smaller natural frequencies for every value of n.

Thus the static load exerts the greatest influence on the amplitude of forced oscillations in the case when the frequency of the forcing dynamic load is close to the lower characteristic frequency of the shell for every value of n. This agrees well with experimental results.

The results obtained show that, in a number of cases, the static load exerts a substantial influence on the amplitude of the forced oscillations of the shell. This confirms the fairly high sensitivity of the amplitude-frequency relations of shells of zero curvature towards static loads, and this property can be made use of in various technical applications.

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